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Degree correlations in the group preferential model

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Abstract

We focus on the study of degree correlations of the group preferential model, in which a link is established with a pre-existing vertex according to an attachment probability which depends on the degree of the m targeted vertices of the new vertex. By using the rate equation approach, we have obtained asymptotic expressions for two vertex and three vertex degree correlations. Finally, all analytical solutions are successfully contrasted with computer simulations.

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1. Introduction

The last decade has witnessed the birth of a new movement of interest and research in the study of complex networks, i.e. networks whose structure is irregular, complex and dynamically evolving in time, with the main focus moving from the analysis of small networks to that of systems with thousands or millions of nodes, and with a renewed attention to the properties of networks of dynamical units (see [1–6], and references therein).

The distribution of degrees $p(k)$ was the first feature for which network models were developed. Plenty of models were proposed for generating the so-called scale-free networks, webs of interactions whose degree distribution approximates a power law, $p(k) \sim k^{-\gamma}$. It completely determines the statistical properties of uncorrelated networks.

However, a large number of real networks are correlated in the sense that a node of degree k is connected to another node of degree, say k', depend on k. This is the case, for instance, in many social networks [7], the degree correlations are positive, and nodes with a high degree tend to connect to each other. Conversely, in protein interaction networks [8, 9], or the Internet [10], the degree correlations are negative, highly connected nodes avoid linking directly to each other and instead connect to low-degree ones. Hence, in these cases, it is necessary to introduce the conditional probability $P(k'|k)$, being defined as the probability that a link from

a node of degree k points to a node of degree k'. Although the degree correlations are formally characterized by $P(k'|k)$, the direct evaluation of the conditional probability gives extremely noisy results for most of the real networks because of their finite size N. This problem can be overcome by defining the average nearest neighbors degree of a node i as

$$k_{nn,i} = \frac{1}{k_i} \sum_{j \in \mathcal{N}_i} k_j, \tag{1}$$

where the sum runs on the nodes belonging to $\mathcal{N}_i$, the set of first neighbors of i. By using the definition (1), one can calculate the average degree of the nearest neighbors of nodes with degree k, denoted as $k_{nn}(k)$, obtaining an expression that implicitly incorporates the dependence on k [11].

Analogously to two vertex correlations, correlations implying three vertices can be measured by means of the probability $P(k', k''|k)$ that a vertex of degree k is simultaneously connected to vertices of degree k' and k''. Again, the difficulties in directly estimating this conditional probability can be overcome by analyzing the clustering coefficient. The average clustering coefficient of the vertices of degree k (the clustering spectrum), $\bar{c}(k)$, can be formally computed as the probability that a vertex of degree k is connected to vertices of degree k' and k'', and those two vertices are at the same time joined by an edge, averaged over all the possible values of k' and k''.

Recent studies have focused on a more detailed topological characterization of the degree correlations among vertices. Barrat and Pastor-Satorras [12] proposed a rate equation approach to compute two vertex correlations in scale-free growing network models based on the preferential attachment mechanism. They studied the properties of both two and three vertex correlations for linear preferential attachment models and also for a model yielding a large clustering coefficient. Later, Garcia-Domingoa *et al* [13] studied the degree distribution and the two-node degree correlations in growing networks generated via a general linear preferential attachment of new nodes together with a uniformly random deletion of nodes by using a continuum approach.

Kong *et al* [14] newly introduced a new idea and proposed a group preferential model which takes the m targeted edges of a new node as a unity. By using the Markov chain theory, we prove that the group preferential model is of scale free, and a explicit analytical formula for the degree distribution is provided.

For a deeper understanding of the structure and function of the group preferential model, in this paper, we further offer a complete study of the two-node degree correlation and three-node degree correlation. The remainder of the paper is organized as follows. In section 2, we first recall the model and gain the expression of degree $k_i(t)$ by using the mean-field approach. In sections 3 and 4, the two vertex and three vertex degree correlations are calculated, respectively. Finally, the analytical results are checked by numerical simulations in section 5.

2. Preliminaries

In this section, we will introduce some notations, the model and the expressions of degree distribution.

Let us introduce some notations. In a network $G_t = \{-n_0 + 1, \ldots, -1, 0\} \cup \{1, 2, 3, \ldots, t\}$, denote the total number of vertices and the total degree by n_t and N_t, respectively. We name each new vertex by the time it enters the network and call a combination of any m vertices $(i_1, i_2, \ldots, i_m)$ in the existing network a group, denoted by $g(i_1, i_2, \ldots, i_m)$.

2.1. The model

In the following, we recall the exact description of the model discussed in this paper. *Group preferential model (GP model)*

(i) *Initial condition*: start with a small number of n_0 nodes with total degree N_0 and denote the n_0 initial nodes as $\{-n_0+1,\ldots,-1,0\}$.

(ii) *Growth*: at each time step, add a new node with $m(1 \leqslant m \leqslant n_0)$ new edges that connect the new node to m different old vertices $(i_1, i_2, \ldots, i_m)$ with preferential attachment probability $\Pi(k_{i_1}, k_{i_2}, \ldots, k_{i_m})$. In particular, the rule for a new node to choose its m targeted vertices is as follows: first, compute the number of possible groups of G_t, namely the number of possible m-element combinations of G_t. Then, choose a group and make connections with each node in it. The probability that a new node points to a group is proportional to the sum of the node degree in that group.

Hence, from the model description above, a graph G_t can be separated into $C_{n_t}^m$ groups. And the probability $\Pi\big(k_{i_1}, k_{i_2}, \ldots, k_{i_m}\big)(t)$ that a new vertices $t+1$ connects to a group $g(i_1, i_2, \ldots, i_m)$, is

$$\Pi\big(k_{i_1}, k_{i_2}, \ldots, k_{i_m}\big)(t) = \frac{k_{i_1}(t) + k_{i_2}(t) + \cdots + k_{i_m}(t)}{C_{n_t-1}^{m-1} N_t}. \tag{2}$$

The denominator on the right-hand side of equation (2) is the total degree of nodes in the sample space. And the sample space here accounts for all the possible m-element combinations in graph G_t, which relates to the set of all possible outcomes in classical sense.

Remark. In the GP model, each new node t takes the existing network as $\binom{n_t-1}{m}$ possible groups, then chooses one to make connections among the elements in it. Additionally, the probability for a new node to establish connection with an old one is proportional to the sum of the degree of its m targeted vertices, not just the single one as in common cases, which will ultimately improve the attachment probability for low-degree nodes.

For a vertex i, let $\Pi(k_i(t))$ denote the probability that it receives an edge from a new vertex $t+1$. Note that it is equal to the total attachment probability of groups which include node i, then we have

$$\Pi(k_i(t)) = \sum_{i_1<\cdots<i_{m-1}\in\{1,2,\ldots,n_t\}/\{i\}} \Pi\big(k_i, k_{i_1}, k_{i_2}, \ldots, k_{i_{m-1}}\big)(t).$$

Following the knowledge of combination, k_i and each $k_{i_s}(i_s \in G_t/\{i\})$ are computed repeatedly $C_{n_t-1}^{m-1}$ and $C_{n_t-2}^{m-2}$ times, respectively, one can derive that

$$\begin{aligned}\Pi(k_i(t)) &= \frac{C_{n_t-1}^{m-1} k_i(t) + C_{n_t-2}^{m-2} \sum_{j\neq i} k_j(t)}{C_{n_t-1}^{m-1} N_t} \\ &= \frac{n_t - m}{n_t - 1}\frac{k_i(t)}{N_t} + \frac{m-1}{n_t-1}.\end{aligned}$$

Hence, we can get easily

$$\begin{aligned}\Pi(k_i(t)) &= \frac{n_0 + t - m}{n_0 + t - 1}\frac{k_i(t)}{2mt + N_0} + \frac{m-1}{n_0+t-1} \\ &\simeq \frac{k_i(t)}{2mt} + \frac{m-1}{t}.\end{aligned} \tag{3}$$

For large values of t, we also approximate that

$$n_t \simeq t, \qquad N_t \simeq 2mt.$$

2.2. Degree distribution

From the evolution rules (i), (ii) and equation (3), one gets that the degree evolution is governed by the following rate equation:

$$\frac{\partial k_i(t)}{\partial t} = \frac{k_i(t)}{2mt} + \frac{m-1}{t}, \tag{4}$$

with the initial condition $k_i(i) = m$. Integrating equation (4), one obtains

$$k_i(t) = m(2m-1)\left(\frac{t}{i}\right)^{\beta} - 2m(m-1), \tag{5}$$

where $\beta = \frac{1}{2m}$.

Therefore, this model yields networks with a power-law degree distribution of the form

$$P(k) = 2m(2m^2-m)^{2m+1}(k+2m(m-1))^{-(2m+1)} \tag{6}$$

$$\sim k^{-\gamma}, \quad \gamma = 2m+1, \qquad \text{for large } k. \tag{7}$$

In the following two sections, we turn to discuss the two vertex and three vertex degree correlations by the rate equation approach.

3. Two vertex degree correlations

In this section, we will start by following the rate equation approach for obtaining an analytical expression for two vertex degree correlations in the GP model.

The average degree of the nearest neighbors of a node s at time t, $\bar{k}_{nn}(s,t)$, is a function of its degree, $k_s(t)$. By definition,

$$\bar{k}_{nn}(s,t) = R_s(t)/k_s(t),$$

where $R_s(t)$ is the sum of the degrees of the neighbors of the node s at time t. That is, $R_s(t) = \sum_{j\in v(s)} k_j(t)$, where $v(s)$ denotes the neighbors of s.

From the expression for $\bar{k}_{nn}(s,t)$, we need to obtain previously the time evolution of two important features of the network, namely, $R_s(t)$ and the variance of the degree distribution.

Consider a fixed node s. Let us derive a difference equation for the time evolution of R_s. The entering node attaches to s with probability $\Pi(k_s)$, and in this case R_s increases by m. This new node attaches to a neighbor of s with probability $\sum_{j\in v(s)} \Pi(k_j)$. As usual, we will not consider the fact that the entering node attaches simultaneously to s and to one or several neighbors of s, since the probability for this event is negligible when the size of the network is large. Summarizing, we get the following rate equation for the evolution of $R_s(t)$:

$$\begin{aligned}\frac{\mathrm{d}R_s(t)}{\mathrm{d}t} &= m\Pi(k_s(t)) + \sum_{j\in v(s)} \Pi(k_j(t)) \\ &= \frac{R_s(t)}{2mt} + \frac{2m-1}{2t}k_s(t) + \frac{m(m-1)}{t} \\ &= \frac{R_s(t)}{2mt} + \frac{m(2m-1)^2}{2t}\left(\frac{t}{s}\right)^{\beta-1} - \frac{2m(m-1)^2}{t}.\end{aligned}$$

The general solution of the previous equation is

$$R_s(t) = f_0(s)t^{\beta} + \beta[m(2m-1)]^2\left(\frac{t}{s}\right)^{\beta} lnt + [2m(m-1)]^2. \tag{8}$$

where $f_0(s)$ is given by the boundary condition $R_s(s)$, that is

$$R_s(s) = \sum_{j=1}^{s} \Pi(k_j(s))[k_j(s)+1] = \frac{\sum_{j=1}^{s} k_j^2(s)}{2ms} + m(2m-1).$$

Plugging $k_i(t) = m(2m-1)\left(\frac{t}{i}\right)^{\beta} - 2m(m-1)$ into $R_s(s)$ results in

$$R_s(s) = m[1-2m(m-1)] + \beta[m(2m-1)]^2 s^{2\beta-1} \sum_{j=1}^{s} j^{-2\beta}. \tag{9}$$

The summation in equation (9), $\sum_{j=1}^{s} j^{-2\beta} \simeq s^{1-2\beta}/(1-2\beta)$, is convergent, and therefore $R_s(s)$ becomes independent of s. This leads to

$$R_s(t) \simeq \beta[m(2m-1)]^2 \left(\frac{t}{s}\right)^{\beta} ln\left(\frac{t}{s}\right) + \left(m[1-2m(m-1)] + \frac{\beta[m(2m-1)]^2}{1-2\beta} - (2m(m-1))^2\right)\left(\frac{t}{s}\right)^{\beta} \tag{10}$$

and finally the dominant behavior for the correlation function is

$$\bar{k}_{nn}(k, N) \simeq \frac{2m-1}{2} ln\left(\frac{k}{m(2m-1)}\right). \tag{11}$$

In this case, $\bar{k}_{nn}(k, N)$ is independent of the network size and increases logarithmically with k. The group preferential attachment yields networks with assortative degree correlations.

4. Three vertex degree correlations

Recall that the clustering coefficient $c_s(t)$ of vertex s at time t is defined as the ratio between the number of edges, the neighbors of s and its maximum possible value. Then, if $M_s(t)$ is the number of connections between the neighbors of s at time t, we have that

$$c_s(t) = \frac{2M_s(t)}{k_s(t)(k_s(t)-1)}. \tag{12}$$

During the growth of the network, $M_s(t)$ can only increase by the simultaneous addition of an edge to s and one of its neighbors. Let $\Pi(k_s(t), k_i(t))$ be the probability that vertices s and i are connected simultaneously by a node. Similarly, it equals the total attachment probability of groups which include nodes s and i, then we have

$$\Pi(k_s(t), k_i(t)) = \sum_{i_1<\cdots<i_{m-1}\in\{1,2,\ldots,n_t\}/\{s,i\}} \Pi\left(k_s, k_i, k_{i_1}, k_{i_2}, \ldots, k_{i_{m-2}}\right)(t).$$

Following the knowledge of combination, k_s, k_i and each $k_{i_s}(i_s \in G_t/\{s,i\})$ are computed repeatedly $C_{n_t-2}^{m-2}$ and $C_{n_t-3}^{m-3}$ times, respectively, one can derive that

$$\Pi(k_s(t), k_i(t)) = \frac{C_{n_t-2}^{m-2}(k_s(t)+k_i(t)) + C_{n_t-3}^{m-3}\sum_{j\neq s,i} k_j(t)}{C_{n_t-1}^{m-1} N_t} = \frac{m-1}{(t-1)(t-2)}\left(m-2+\frac{t-m}{2mt}(k_s(t)+k_i(t))\right). \tag{13}$$

Hence, the probability of the simultaneous addition of an edge to s and one of its neighbors is

$$\sum_{i\in v(s)} \Pi(k_s(t), k_i(t)) = \frac{m-1}{(t-1)(t-2)}\left((m-2)k_s(t) + \frac{t-m}{2mt}\left(k_s^2(t) + R_s(t)\right)\right) \simeq \frac{m-1}{t^2}\left((m-2)k_s(t) + \frac{1}{2m}\left(k_s^2(t) + R_s(t)\right)\right). \tag{14}$$

Therefore, in the continuous k approximation, we can write down the following rate equation:

$$\frac{\mathrm{d}M_s(t)}{\mathrm{d}t} = \frac{m-1}{t^2}\left((m-2)k_s(t) + \frac{1}{2m}\left(k_s^2(t) + R_s(t)\right)\right). \tag{15}$$

In order to solve this equation, we approximate $k_s(t)$ and $R_s(t)$ by their dominant terms for large t and s,

$$k_s(t) \simeq m(2m-1)\left(\frac{t}{s}\right)^{\beta}, \qquad R_s(t) \simeq \beta[m+2m(m-1)]^2\left(\frac{t}{s}\right)^{\beta} ln\left(\frac{t}{s}\right),$$

yielding

$$\begin{aligned} M_s(t) = (m-1)\Bigg(&\frac{(m-2)m(2m-1)}{\beta-1}s^{-\beta}(t^{\beta-1}-s^{\beta-1}) \\ &+\frac{m(2m-1)^2}{2(\beta-1)}s^{-\beta}\left[t^{\beta-1}ln\left(\frac{t}{s}\right)-\frac{1}{\beta-1}(t^{\beta-1}-s^{\beta-1})\right] \\ &+\frac{m(2m-1)^2}{2(2\beta-1)}s^{-2\beta}(t^{2\beta-1}-s^{2\beta-1})\Bigg) + M_s(s). \end{aligned} \tag{16}$$

Next, we turn to compute the boundary condition $M_s(s)$. To do so, we observe that $M_s(t)$ is the number of triangles created by the introduction of vertex s. Therefore

$$M_s(s) = \frac{1}{2}\sum_{j,n}\Pi(k_j(s), k_n(s))\Pi_{j,n}, \tag{17}$$

that is, it is proportional to the probability that s is connected to vertices j and n, times the probability $\Pi_{j,n}$ that j and n are linked, averaged over all vertices j and n existing in the network at time s. The probability $\Pi_{j,n}$ is given by

$$\Pi_{j,n} = \Theta(j-n)\Pi(k_n(j)) + \Theta(n-j)\Pi(k_j(n)),$$

where $\Theta(x)$ is the Heaviside step function.

The boundary condition $M_s(s)$ can be written as

$$\begin{aligned} M_s(s) &\simeq \frac{1}{2}\sum_{j,n}\frac{m-1}{s^2}\left(\frac{1}{2m}(k_j(s)+k_n(s))\right) \\ &\quad\times(\Theta(j-n)\Pi(k_n(j)) + \Theta(n-j)\Pi(k_j(n))) \\ &= \frac{(m-1)(2m-1)^2}{8}s^{\beta-2}\left(\sum_{n=1}^{s}n^{-\beta}\sum_{j=n+1}^{s}j^{-1} + \sum_{n=1}^{s}n^{-2\beta}\sum_{j=n+1}^{s}j^{\beta-1}\right. \\ &\quad\left.+\sum_{j=1}^{s}j^{-2\beta}\sum_{n=j+1}^{s}n^{\beta-1} + \sum_{j=1}^{s}j^{-\beta}\sum_{n=j+1}^{s}n^{-1}\right) \\ &= \frac{(m-1)(2m-1)^2}{4}s^{\beta-2}\left(\sum_{n=1}^{s}n^{-\beta}\sum_{j=n+1}^{s}j^{-1} + \sum_{n=1}^{s}n^{-2\beta}\sum_{j=n+1}^{s}j^{\beta-1}\right). \end{aligned} \tag{18}$$

We observe that the double summations in equation (18) take the forms at large s, respectively,

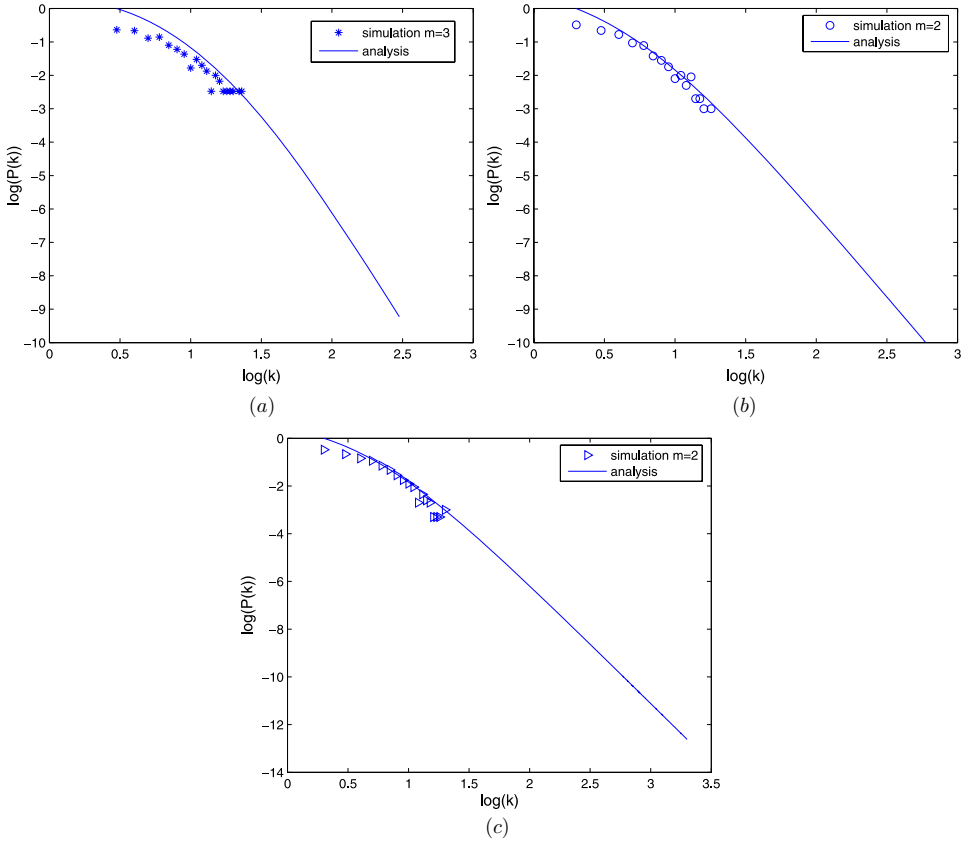

Figure 1. Comparison between simulation and analytic results for degree distributions with system sizes $t = 300, m = 3$ and $t = 1000, 2000, m = 2$ in logarithmic scales. Symbols correspond to the different system sizes $t = 300$ (*), 1000 (○), 2000 (▷). (*a*) $t = 300, m = 3$; (*b*) and (*c*) $t = 1000, 2000, m = 2$, respectively. The solid lines correspond to the theoretical value (equation (6)), representing a power-law decay with exponent $-(2m + 1)$.

$$\Gamma_1 = \sum_{n=1}^{s} n^{-\beta} \sum_{j=n+1}^{s} j^{-1} \simeq \sum_{n=1}^{s} n^{-\beta}(lns - lnn) \simeq \frac{s^{1-\beta}}{(1-\beta)^2}, \tag{19}$$

$$\Gamma_2 = \sum_{n=1}^{s} n^{-2\beta} \sum_{j=n+1}^{s} j^{\beta-1} \simeq \sum_{n=1}^{s} n^{-2\beta}\beta^{-1}(s^{\beta} - n^{\beta}) = \beta^{-1}[\zeta(2\beta)s^{\beta} - \zeta(\beta)]. \tag{20}$$

Solving the equation for $M_s(t)$ with the boundary condition (equation (17)), thus we obtain

$$\begin{aligned} M_s(t) = (m-1)\Bigg(& \frac{(m-2)m(2m-1)}{\beta-1} s^{-\beta}(t^{\beta-1} - s^{\beta-1}) \\ & + \frac{m(2m-1)^2}{2(\beta-1)} s^{-\beta}\left[t^{\beta-1} ln\left(\frac{t}{s}\right) - \frac{1}{\beta-1}(t^{\beta-1} - s^{\beta-1}) \right] \\ & + \frac{m(2m-1)^2}{2(2\beta-1)} s^{-2\beta}(t^{2\beta-1} - s^{2\beta-1}) \Bigg) \\ & + \frac{(m-1)(2m-1)^2}{4} s^{\beta-2}\beta^{-1}\left(\frac{\beta s^{1-\beta}}{(1-\beta)^2} + \zeta(2\beta)s^{\beta} - \zeta(\beta) \right). \end{aligned} \tag{21}$$

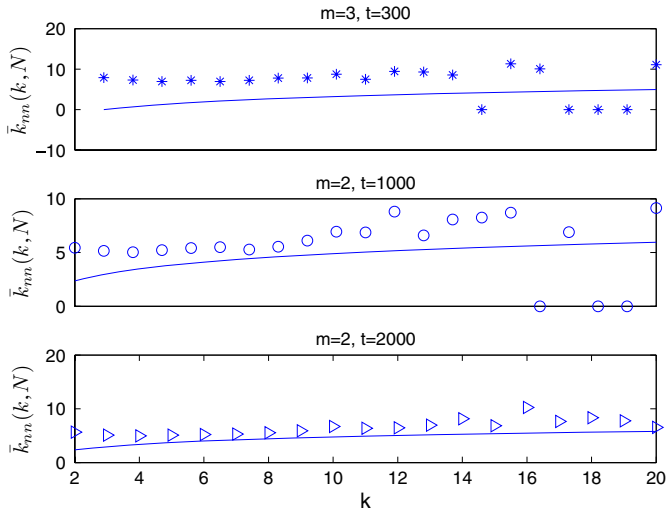

Figure 2. Average degree for the nearest neighbors of the nodes of degree k, $\bar{k}_{nn}(k, N)$, for the GP model with system sizes $t = 300, m = 3$ and $t = 1000, 2000, m = 2$. The solid lines correspond to the theoretical value (equation (11)) and symbols correspond to the different system sizes $t = 300$ (*), 1000 (○), 2000 (▷). Top plots: $t = 300, m = 3$. Middle and bottom plots: $t = 1000, 2000, m = 2$, respectively.

Since in growing network models in the continuous k approximation the degree at time t is uniquely determined by the introduction time s, from $c_s(t)$ we can directly obtain the clustering spectrum $\bar{c}(k, N)$ as a function of k and the largest time $t = N$.

Hence, in order to express explicitly the dependence of $c_s(t)$ on the vertex degree, we write s in terms of k and N, that is

$$\bar{c}(k, N) \simeq \frac{(m-1)(2m-1)^2[m(2m-1)]^{2-\frac{2}{\beta}}}{4}\beta^{-1}\zeta(2\beta)k^{\frac{2}{\beta}-2}N^{2\beta-2}. \qquad (22)$$

Therefore, we obtain that the average clustering of the vertices of the degree k is a growing function of k, scaling as $\bar{c}(k, N) \sim k^{\frac{2}{\beta}-4}N^{2\beta-2}$. Since by definition the clustering must be smaller than 1, this growing behavior must be restricted to degree values $k \lesssim N^{\beta}$.

5. Numerical simulation

In order to check the analytical results obtained in this paper, we have performed extensive numerical simulations of the GP model, such as the degree distribution, the degree correlation and the clustering coefficient. Simulations of the model consistently confirm the analytical results obtained in the previous sections.

Figure 1 displays the results of the numerical exploration of degree distribution, which shows that the data follow the predicted scaling $P(k) \sim k^{-(2m+1)}$ as the network increases.

In figure 2, the numerical explorations correspond to the average degree of the nearest neighbors as a function of the degree, $\bar{k}_{nn}(k, N)$. We observe that, as the size of the network increases, the data follow the predicted function independent of the network size, and slowly (logarithmically) growing with the degree k, consequently assortative by degree.

Finally, in figure 3 we show the numerical exploration of the clustering spectrum as a function of node degree for different system sizes. The solid lines correspond to the theoretical

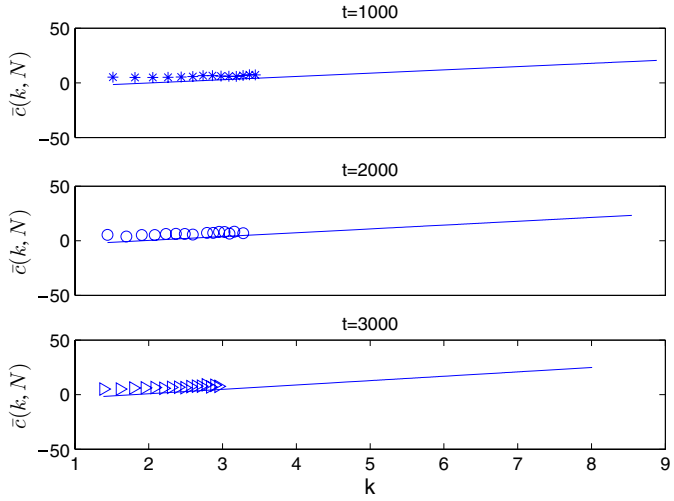

Figure 3. Comparison between simulation and analytic results for clustering spectrum $\bar{c}(k, N)$ for the GP model with system sizes $t = 1000, 2000, 3000, m = 2$ in logarithmic scales. The simulation data are rescaled by the size prefactor $N^{2-2\beta}$. The solid lines correspond to the theoretical value (equation (22)) and symbols correspond to the different system sizes $t = 1000$ (*), 2000 (○), 3000 (▷).

value (equation (22)). The simulation data are rescaled by the size prefactor $N^{2-2\beta}$, showing the dependence with the system size, due to the finite size effect.

6. Conclusion

The GP model considered in this paper has introduced a new idea in the research of the complex network, which takes the existing network as many groups (each with size m). And in this paper, we have shown that it is scale free and further discussed its degree correlations.

Based on the rate equation in the continuous k approximation, together with appropriate boundary conditions, we have obtained asymptotic expressions for the two vertex degree correlation $\bar{k}_{nn}(k, N)$ and the three vertex degree correlation $\bar{c}(k, N)$. In particular, the two vertex correlations here is expressed by means of the average degree of the nearest neighbors of the vertices of degree k, $\bar{k}_{nn}(k, N)$. Additionally, we have presented a more complete description of the rate equation determining the cluster spectrum $\bar{c}(k, N)$. Furthermore, as shown in figures 1–3, we can observe that there is an overall good agreement between simulated data and theoretical results, including the degree distribution, the degree correlation and the clustering spectrum.

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